CHAPTER 16

Plato's Influences in the Greek Religious Music Theory

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In the *Timaeus*, Plato presents the Creator who "gave the world the shape 'which was suitable and akin to its nature,' i.e., a rounded spherical shape, 'because he believed that likeness is incalculably more excellent than unlikeness' (33b). The Creator then assigned to the world a rotatory or circular movement, 'which is the most appropriate to mind and intelligence' (34a);"[1] Timaeus in particular mentions:

> Now the making of the universe took up the whole bulk of each of these four elements. Of all fire and all water and air and earth its framer fashioned it, leaving over no part nor power without. Therein he had this intent […] that it might be a creature perfect to the utmost with all its parts perfect […] And he assigned to it its proper and natural shape. To that which is to comprehend all animals in itself that shape seems proper which comprehends in itself all shapes that are. Wherefore he turned it in a rounded and spherical shape, having its bounding surface in all points at an equal distance from the centre: this being the most perfect and regular shape; for he thought that a regular shape was infinitely fairer than an irregular […] For he assigned it that motion which was proper to its bodily form, of all the seven [: "up and down, forwards and backwards, to right and to left, and finally rotation upon an axis"] that which most belongs to reason and intelligence. Wherefore turning it about uniformly in the same spot on its own axis, he made it to revolve round and round; but all the six motions he took away from it and left it without part in their wanderings.[2]

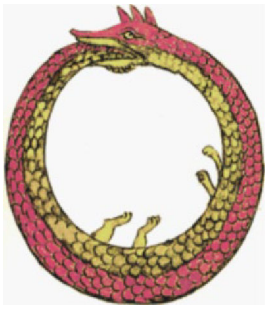
This essay centres on the symbol of the circle, a shape that symbolizes the universe which surrounds us; it reflects the harmony of the world in which we live, a world that is driven by circular movements. The principle

[1] Kalfas 2005, 164.
[2] Archer-Hind 1888, 101–103.

of 'circularity' is "a key element in the Pythagorean view of the world" and is foregrounded in Plato's work wherein it "becomes a predominant pattern in platonic astronomy."[3] At the same time, the circle is a symbol of crucial importance for the Greek Religious Music Theory as well; through a circle, the so-called *Wheel of the Eight Modes*, the masters of the Greek Religious Music used to teach the Theory of the same Music; one can grasp the philosophical background and the inmost symbolism associated with this circular shape's ancient use, reflected in the inner sense of the entire Greek Religious Music.

In fact, the circle represents one of the most ancient symbols, undoubtedly polysemous, which have led to intellectual quests from time immemorial.[4] It symbolizes rejuvenation, the return to unity through successive transfigurations, eternity. Its par excellence depiction was through the shape of an ancient symbol called 'Ouroboros,' a serpent which coils around itself and eats (or devours) its own tail:

The 'Ouroboros'.[5]



The 'Ouroboros' (detail).[6]

[3] Kalfas 2005, 163–164 [about *platonic astronomy* in general see Kalfas 2005, 151–174].

[4] In onwards writings, a material found in the following two sites has been used: Ουροβόρος όφις - Βικιπαίδεια [31-7-2018]; & http://www.artofwise.gr/html/categoriescontent/symvola/ourovoros.html [31-7-2018].

[5] An image from Greek Alchemy Manuscript, taken from the above mentioned sites.

[6] A detail of the same image from Greek Alchemy Manuscript, taken from the above mentioned sites.

The circle is one of the oldest mystical symbols, which has been used widely both in Gnosticism (for example, the Gnostic sects Naassenes and Ophites) and Freemasonry; a symbol representing wisdom, perfection, the creative impetus of every new start, rebirth, as well as infinity; the circle represents the concept of a constant and eternal return, the perpetual circle of life. Later, the Alchemists considered the 'Ouroboros' as representing "the unity of all," that is, the unity of matter, which symbolizes the soul of the world, and thus stands for the unity of all things, the cycle of life and death. Another mystical saying often found inside the said symbol or on the body of the 'Ouroboros' is: "all is one and in my beginning is the end":

The 'Ouroboros'.[7]

The circle, as commented earlier by the Greek Scholar Simon Karas,[8] outlines the "universal harmony, where […] the most inferior and the weakest beings are encircled by the most divine and powerful;" Karas continues: "The perfect expression and representation of this universal harmony, which is seen through the use of ratios of the nine numbers of the decad – a receptacle symbolizing everything that exists – and their connection to musical intervals [….], is given by the Mathematical Wheel of Theon of Smyrna, a Pythagorean Philosopher. This wheel was studied by our medieval ancestors, who as scholars and followers of the old tradition and theory defined, on the basis of this wheel, the development of the eight modes system, which is divided into two pairs of four modes, diagonally sketched, also known as the Wheel of the Octaechia:"

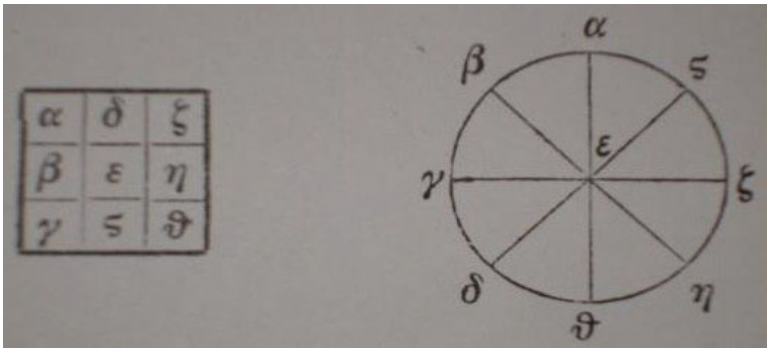
[7] An image from Codex Marcianus (11th c.), taken from the above mentioned sites.
[8] Karas 1970, 22–23.

The Mathematical Wheel of Theon of Smyrna &
the *Wheel of the Eight Modes* of the Greek Religious Music.[9, 10]

This specific wheel of Theon of Smyrna – which is described in his known work *Mathematics Useful for Understanding Plato,* in particular in the second part of his book, in the Chapter "Properties of the Numbers Contained in the Decad (*Tetraktys*)" – represents a diagram which results from the following comment of number five:

> Number 5 is the mean term of (two numbers whose sum is) the decad, because if, by the addition of any two numbers, one obtains ten, the means of these numbers will be 5 according to arithmetic proportion. Thus, for example, if you add 9 and 1, 8 and 2, 7 and 3, 6 and 4, the sum will always be 10 and the mean term in arithmetic proportion will be 5. This is shown in the diagram in which any addition of two opposite numbers gives 10, the proportional arithmetic mean being 5, which is greater than one of the extremes and less than the other, by the same difference. This number is also the first which embraces the two types of numbers, the odd and the even, that is, 2 and 3, for unity is not a number:[11]

[9] Karas 1982, 233.
[10] Karas 1970, 40.
[11] Lawlor/Toulis 1979, 67.



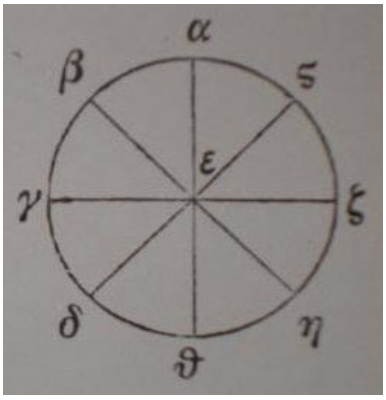
Diagrams of the Mathematical Wheel & Square of Theon of Smyrna.[12]

1	4	7
2	5	8
3	6	9

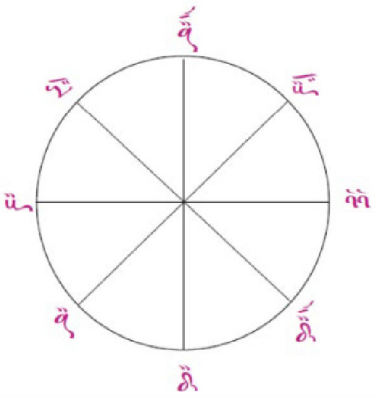
Diagram of the Mathematical Square of Theon of Smyrna.[13]

The described Mathematical Wheel of Theon of Smyrna, i.e., the aforementioned diagram (where 5 is the mean number of 10, "according to arithmetic proportion," the number "which embraces the two types of even and odd numbers," the number "which is greater than one of the extremes and less than the other, by the same difference"), this diagram is the same exact shape which was predominantly and repeatedly used (since the 14th century at least and onward) for both supervisory and educational teaching of the basic data of the Theory of the Greek Religious Music:

[12] Hiller 1878, 102.
[13] Lawlor/Toulis 1979, 67.



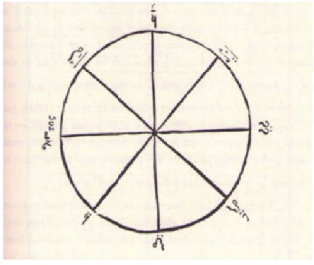
The Mathematical Wheel of Theon of Smyrna.

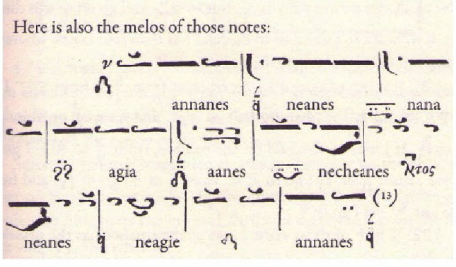


The *Wheel of the Eight Modes* of the Greek Religious Music.

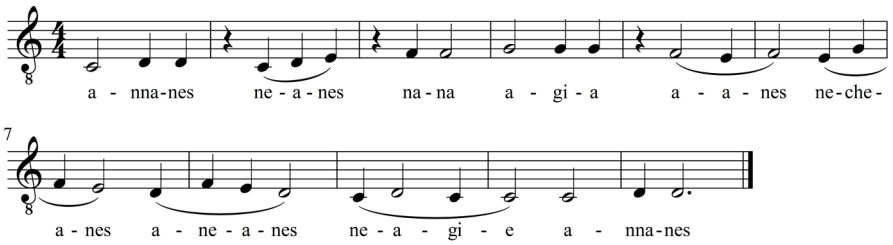
This is a reference to the above-mentioned *Wheel of the Eight Modes*, a renowned shape in the Theory of the Greek Religious Music, which is often depicted in manuscripts or published books, a symbol which is commented thoroughly by the theoretical pioneer of this music, Chrysanthos of Madytos.[14] Aside from any other specific reference to the

[14] Chrysanthos 2010, 56-60[§§66-76]. Chrysanthos (who remains the major and basic tutor of the Theory of the Greek Religious Music until today) quotes the common scheme of the said *Wheel*, also elucidating that through it one can ascend and descend 'diatonically' (that is, according to the so-called Diatonic Genus of the Greek Religious Music Modes) the intervals of a 'pentachord' (i.e., a fifth), following a particular melodic development:





See Chrysanthos 2010, 57[§68]; a corresponding transcription into Staff Notation of the above musical example – written in Byzantine Notation – could be formed as follows:



More specifically, Chrysanthos comments on:

a. The name of the *Wheel* (p. 56[§§67&66]):

> "The ecclesiastical musicians call *trochos* a method, by which they ascend and descend diatonically the intervals of the pentachord, with the eight words, or polysyllables notes; in ascent: *annanes*, *neanes*, *nana*, *agia*; in descent: *aanes*, *necheanes*, *aneanes*, *neagie*. Those eight notes are called the notes of trochos."

b. The shape and the construction of the *Wheel* (p. 56[§68]):

> "The trochos is constructed if in any circle four diameters intersect each other; and on the end of one is written ; and on the end of the next, ; and on the end of the following, ; and on the end of the fourth, ; and then, on the opposite end of the first diameter is written ; of the second, ; of the third, ; of the fourth, ."

c. The way to read the *Wheel* musically (p. 58[§69]):

> "We chant those notes on the trochos, starting from and pronouncing it *annanes*, with the melos represented by its neumes; then coming upon we pronounce it *neanes* with its melos; then coming upon we pronounce it *nana* with its melos; then we come on and pronounce it *agia* with its melos; and turning to the other end, we find and pronounce it *aanes* with its melos; then going to we pronounce it *necheanes* with its melos; going then to we pronounce it *aneanes* with its melos; arriving on we pronounce it *neagie* with its melos and from this point, we turn to the other end, and finding , we pronounce it *annanes* as said before."

d. The expansion of the interval data of the *Wheel*; through the observation that in order to expand the vocal ascent or descent, a constant repetition (or, better, a circulation) of the following data is simply required (p. 58[§70]):

> "This way, we chant four notes in ascent and four in descent. But if we wish to ascend more, then, coming to , we do not turn to , but to and advance. And if we wish to descend more, when we arrive to we do not turn to , but to and go on to this direction."

e. The created on the *Wheel* associations between the vocal ascent and descent; through the observation that at the opposite end of every note of the ascent, the corresponding note of the descent can be found (and vice versa) (p. 58[§§70-1]):

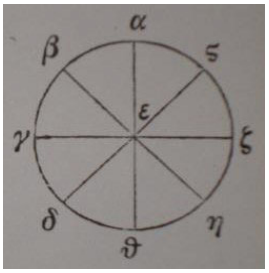
> "If we want to ascend or to descend less than four notes, wherever we happen to

be, we turn to the opposite end. If I wish to ascend on the trochos, from  or from  , I go to  , from or from  , I go to  , from or from  , I go to  , and from or from  , I go to . If I wish to descend on the trochos, from or  , I go to , from or , I go to , from or , I go to , and from or , I go to ."

So, to summarize what Chrysanthos is describing regarding the so-called *Wheel of the Eight Modes of the Greek Religious Music*, especially regarding its aspects and dimensions, one should pay particular attention to the following three points: Firstly; *the shape and the construction of the Wheel*: the so-called *Trochos* is a usual circle, on which four diameters are formed; in this way, eight points are marked on the given circle, four at the right side and four at the left one; the right side, in which four points are marked (points which obviously indicate four musical steps), symbolizes the ascent of any melody; the poetic text, through which one can perform those musical steps (since the Greek Religious Music is exclusively a phonetic one), is made by the aforementioned words, that is *ananes* | *neanes* | *nana* | *agia*; one can follow this ascending musical direction–moving on according to a clockwise motion – by using the mentioned four steps; the aforementioned melodic version of those words formulates both, the so-called 'polysyllables (that is, made by more than one syllable) notes', used as the typical ascending musical steps in the Greek Religious Music Theory until the beginning of the 19th century (specifically until the year 1814), as well as the 'musical identity' of each one of the four authentic modes (*first, second, third* & *fourth*) of the same Music, as they ascend by step; respectively, the left side, in which four points are also marked (points which obviously indicate four musical steps), symbolizes the relevant descent of the given melody; the poetic text, through which one can perform those musical steps (since the Greek Religious Music is exclusively a phonetic one), is made by the aforementioned words, that is *aanes* | *necheanes* | *aneanes* | *neagie*; one can follow this descending musical direction – moving now according to a counterclockwise motion – by using the mentioned four steps; the aforementioned melodic version of those words formulates both, the so-called 'polysyllables (that is, made by more than one syllable) notes', used as the typical ascending musical steps in the Greek Religious Music Theory until the beginning of the 19th century (specifically until the year 1814), as well as the 'musical identity' of each one of the four plagal modes (*third plagal, second plagal, first plagal* & *fourth plagal*) of the same Music, as they descend by step; it has to additionally be noted that, as one can clearly see on the above scheme of the *Wheel of the Eight Modes of the Greek Religious Music*, the mentioned modes – both authentic and plagal – are symbolized through the relevant letters (*α, β, γ, δ*) of the Greek alphabet. Secondly; *the expansion of the Wheel's interval data*: if one wishes to continue the ascending or descending development of any melody (that means, if one likes to ascend or descend more than the initially given four steps) they have to simply repeat the same series of ascending or descending steps; in other words, when one finishes ascending or descending the mentioned four steps, they have to restart immediately the procedure from the very beginning, but musically transposed

separate musicological parameters of the *Wheel of the Eight Modes of the Greek Religious Music*, one should mainly point out the following comparative observations regarding its osmosis with the aforementioned mathematical diagram of Theon of Smyrna, influenced by Plato's work.

In the following shape, found in Hiller's version,[15] one can see that four diameters and numbers from 1 to 10 can be carved in a specific circle, in such a manner that the sum of numbers placed at opposite points of each diameter are always equal to 10. The form of the shape as well as the strong means of 'communication' (between the diameters) of the circle's points, are data which are found fully integrated in the *Wheel of the Eight Modes of the Greek Religious Music*, with some significant differences, the most noticeable being the conversion of the counterclockwise start of Theon's mathematical wheel to a clockwise one.



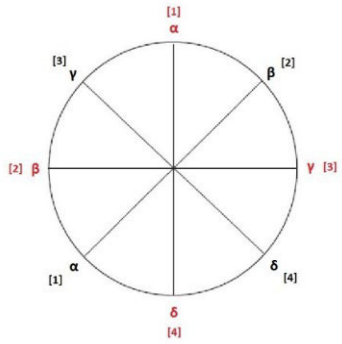
The Mathematical Wheel of Theon of Smyrna.

Moreover, in terms of Arithmology (and given that one is trying to bestow similarly homologous arithmetic data to the *Wheel of the Eight Modes of the Greek Religious Music*), one can notice that it would be possible to use equivalently (through a straight and converse, respectively, perspective) numbers from 1 [α΄] to 4 [δ΄]. During a similar undertaking, one can establish

a fifth higher or lower, respectively. Thirdly; *the diameters of the Wheel, which are creating a simple association and easy connection between vocal ascent and descent*; wherever one happens to be (at any point and step of both the ascent and the descent) they can find and perform the corresponding note (i.e, the previous one of an ascending step or the next one of a descending step, in both cases as far as the development of the musical scale is concerned) exactly at the opposite side of the Wheel.

[15] Hiller 1878, 102.

that there are still latent uses of the aforementioned by Plato crucial numbers, (forming "a perfect expression and representation of the universal harmony") i.e., both 5 (as "the medium which also embraces the two types of even and odd numbers") and 10 ("symbolizing all that exists"); the first one (i.e., the number 5) as the sum of the numbers placed at two opposite points of each two diameters forming a cross inside the circle; and the other one (i.e., the number 10) as the total sum of the numbers placed at four opposite points of the remaining two diameters inside the same circle. Through this observation (according to which the number 5 is developing as a cross in the *middle* of the number 10), it is obvious that there are further hidden, uncommented and unused both mathematical and philosophical correlations between each one of those diagrams in the existing, to this day, bibliography, which should also be explored:



An arithmological overview
of the *Wheel of the Eight Modes* of the Greek Religious Music.

The selection of the circle and the teaching of the Theory of the Greek Religious Music Modes through it is not random at all: along with the shape, its philosophical background, its inmost symbolism was also adopted and transfigured through the prism of the hagiographical saying: "behold, new things have come;"[16] also, in the theological teaching of the so-called "eighth day," which, by being simultaneously the eighth and the

[16] *Corinthians II*, 5:17.

first day, forms again an invisible cycle symbolizing resurrection, eternity, new creation.[17]

[17] Alygizakis 1985, 55–82 (where one can find further bibliography about this matter). Now, one might ask, how does this philosophy reflect on the Theory of the Greek Religious Music Modes or on the general theoretical data of the same Music? Perhaps in a manner that it is indirectly implied in the popular depiction of the *Zodiac Wheel* (a shape with clear similarity to the corresponding *Wheel of the Eight Modes of the Greek Religious Music*) a depiction which (as it is often the case in several orthodox churches) is illustrated on the narthex of the holy monastery of the Dormition of the Mother of God in the area of Rentina in Greece; this specific depiction is completed by the following inscription: "This circle, which is shown before your eyes, is the wheel that revolves all the time, bringing the ones downwards, upwards at great speed […]." (cf. Ezekiel 1929, 34; Kanellos 1993, 28–29):

(Photo taken by Polykarpos Tympas.)

Notice the similarities between the (popular) philosophy of the text of this inscription with everything that is pointed out in the *Great Theory of Music* (Chrysanthos 2010, 32[§72]), at the end of the chapter referring to the *Wheel*; to quote Chrysanthos: "Which of those eight notes of the trochos has the priority? Which comes second, and is it expected to receive its onset from the first? Well, generally speaking, no one, because no one exists without an interval, which implies another note or tone; specifically, though, all of them because every one of them may become first and second etc. depending on one's needs". What a 'democratic' way to describe the nature of the Modes of the Greek Religious Music! In Hagiopolites, another theoretical treatise of the Greek Religious Music, it is pointed out that the "the modes have a marvelous kinship to each other"(Raasted 52[§46]); this point can be appropriately compared to the corresponding quote of Theon of Smyrna, according to which: "by the addition of any two numbers, one obtains ten, the proportional arithmetic mean being 5, which is greater than one of the extremes and less than the other, by the same difference" (Lawlor/Toulis 1979, 67).

However, a question of crucial importance is: how could one understand what the Greek Religious Music Modes means? Usually, these Modes are given specific names according to an arithmetical logic, such as *first*, *second*, *third* & *fourth*; but, such an 'arithmetical name' is only relevant; in a theoretical text (found in No. 872 Vatican Codex) is characteristically written that the names of the Greek Religious Music Modes are "conventionally given";[18] these names could be reformed, in a flexible manner [or should we say, more specifically, in a circular manner], according to the musical circumstances; in another theoretical text, the most ancient relative, dating from the 14th century, the so-called Hagiopolites, we read: "Their names have already been written, both their proper names and those which indicate their order. As far as concerns the Modes, however, it must be born in mind that we do not name the quantity of sounds, but the quality […] Thus, the designations of the Modes are not made for counting purposes but to represent the sound quality of the Melos."[19]

By watching the *Wheel*, one can clearly grasp, both symbolically and in reality, that one Mode presupposes another Mode; one Mode depends on and coexists with the other. The *Wheel* describes, most eloquently, the singular *cyclical* relationship of the Greek Religious Music Modes; those Modes are regularly and perpetually recycled/reused during the course of the melody; and to quote, once again, Hagiopolites: "The above, I suppose, must be enough to demonstrate the marvelous kinship of the Modes. However, if somebody makes an even more accurate scrutiny of these matters, he will find thousands of features which prove the kinship."[20]

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[18] Tardo 1938, 165.

[19] Raasted 1983, 37–38[§30].

[20] Raasted 1983, 59[§55]. Notice that, apart from the above-mentioned narrow consideration of the circle's effect (the implementation of the circular philosophy on the notes and subsequently on the modes), the deepest and broader message of the *Wheel* is reflected through the musical compositions: all Byzantine and post-Byzantine compositions are constructed, planned and developed, based on the same concept, i.e., according to a cyclical motion; the thought of every composer follows a circular movement, while seeking the most appropriate and adequate manner to recycle, in essence, a very limited musical material (constituted by the individual melodic motives) in order to construct a single (short or long) composition. Alygizakis 1985 (81, 82) has already described this phenomenon: "Greek Religious

In simple words, it seems that Greek Byzantine Scholars found a way to set to music, to actually sing, the teachings of Plato's philosophy; the explanations given by Theon of Smyrna through numbers about Plato's musical ideas;[21] a way to sing the *Tetraktys*, which "was the name Pythagoreans gave to the number 10 since it is the product of 1+2+3+4=10."[22] So, nowadays one could attempt such an undertaking, by using either the numbers already used by Plato and Theon, i.e., *Tetraktys'*

Music uses or processes during any musical composition a number of simple musical motives which repeat themselves several times […]. Therefore, Greek Religious Music has its own musical circuit with a stable inner structure and unity of elements on the basis of a well-built coherence. The melodist is limited to the use of eight Modes. Each one of these Modes follows a typical sequence of musical motives, which are bound by a number of technical rules of musical theory in order to create 'a recognizable notion of this mode.' The musical motives are melodic patterns that constitute musical phrases. Thus, a sequence of musical phrases combining a thematic schematization creates a wider musical construction. A specific number of musical phrases or themes constitute the troparion or the hymn."

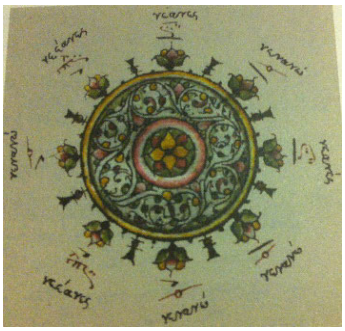
[21] Cf. what is written by Theon of Smyrna, at the "Introduction" of his "Book containing the Numerical Theorems of Music": "Since it is said that there are consonant numbers, the reason for this consonance cannot be found without arithmetic. This consonance has the greatest power, being truth in reason, felicity in life, and harmony in nature; and this harmony, which is diffused throughout the world, will not be found unless it is revealed first through numbers. This harmony, which is also intelligible, is more easily understood than perceptible harmony. We will therefore consider these two harmonies, learning about the one sensed through instruments, and the intelligible harmony which consists in numbers" (Lawlor/Toulis 1979, 32).

[22] Lawlor/Toulis 1979, 39, note 10; cf. also Lawlor/Toulis 1979, 43 (: "The first of all the consonances, says Plato, is the fourth, since it is through it that the others are found; the fifth is only separated from the fourth by the interval of one tone") & 62 (: "Since, as we have shown, all the relationships of the consonances are found in the quaternary [or Tetraktys] of the decad, it is of these numbers that we have to speak. The decad in, fact, constitutes the quaternary, since the sum of the numbers 1, 2, 3, 4 is 10 […]. The importance of the quaternary obtained by addition (that is to say 1, + 2, + 3, + 4) is great in music because all the consonances are found in it. But it is not only for this reason that all Pythagoreans hold it in highest esteem: it is also because it seems to outline the entire nature of the universe. It is for this reason that the formula of their oath was: 'I swear by the one who has bestowed the tetraktys to the coming generations, source of eternal nature, into our souls'. The one who bestowed it was Pythagoras, and it has been said that the tetraktys appears indeed to have been discovered by him").

numbers, or even a series of specific texts, later made by Byzantine Scholars as a sample of polysyllabic (i.e., consisting of more than one syllable) notes, notes actually created through a prayer addressed to God;[23] and it's so impressive to realize that one can do something like that singing either *diatonically* or *chromatically*;[24] at this point, the following relevant

[23] Cf. Chrysanthos 2010, 56[§66], note 11: "The ecclesiastical musicians represented these four intervals, in ascend with the following four words *annanes*, *neanes*, *nana*, *agia*; in descent, with the following, which are similar, *aanes*, *necheanes*, *aneanes*, *neagie*. Those eight notes are called the notes of trochos […]. The *annanes* derives from *ana anes*, that is *anax afes/ἄναξ ἄφες* [*king let*]; the *neanes*, from *ne anes/ναὶ ἄνες* [*do, let*]; the *nana* from *ana ana/ἄναξ ἄναξ* [*king, king*]; and the *agia* from *agie/ἅγιε* [*saint*]; the entire, *anax afes*, *afes*, *anax anax*, *agie/ἄναξ ἄφες*, *ναὶ ἄφες*, *ἄναξ ἄναξ*, *ἅγιε* being a wish addressed to God [*God forgive me, please, forgive me, God God, you, the holy one*]. Constantinos Porphyrogennetos says that the *nana* means *Thee Thee* [*God God*] and the *agia*, *soson di/σῶσον δή* [*do save*]. He also makes up a word composed from those two, *nanaia*."

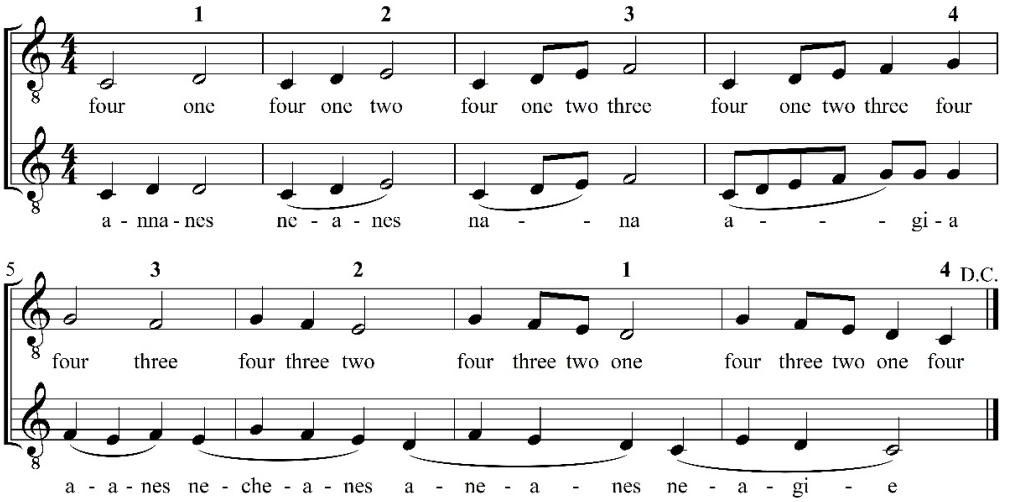
[24] Regarding the 'diatonic version' (that is, the one formed according to the so-called Diatonic Genus of the Greek Religious Music Modes) of the *Wheel of the Eight Modes of the Greek Religious Music*, see what has been already commented above, at the note No. 14. Concerning the relevant *Wheel of the Chromatic Modes of the Greek Religious Music* (which is a similar scheme formed now according to the so-called Chromatic Genus of the Greek Religious Music Modes), see, firstly, the following very interesting illustration of it, published by Konstantinou [6]2013, 334:

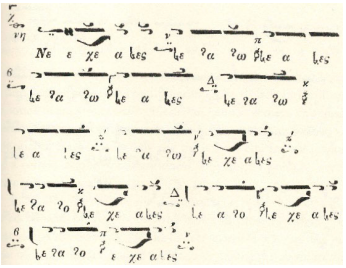


Secondly, cf. what is pointed out in the *Great Theory of Music* (Chrysanthos 2010, 123-125[§§246-52]: "On the Notes of the Chromatic Genus"], where one can also find written down (p. 123[§246]) the following full (ascending and descending) relevant chromatic scale:

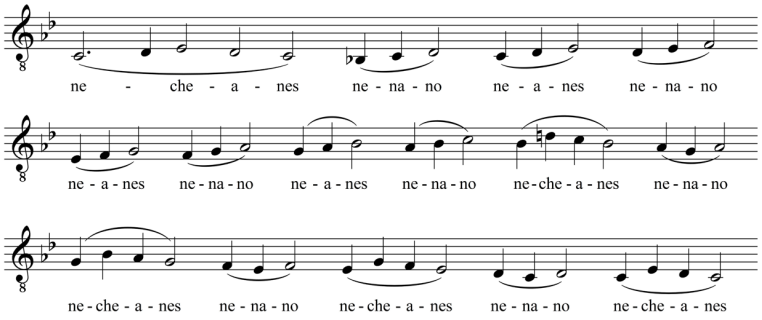
musical examples are, undoubtedly, quite useful:

1. Ascending & Descending 'Diatonic Tetrachord' [Fourth]:

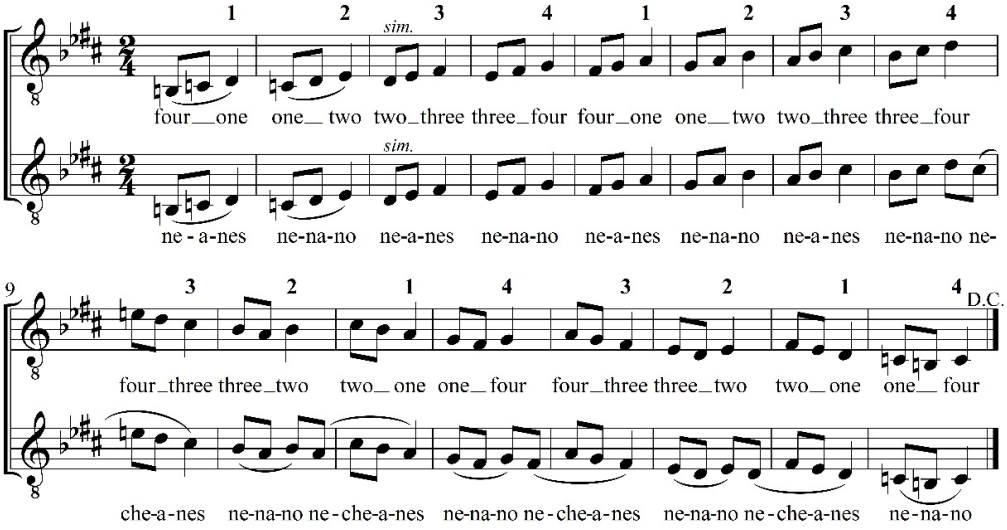




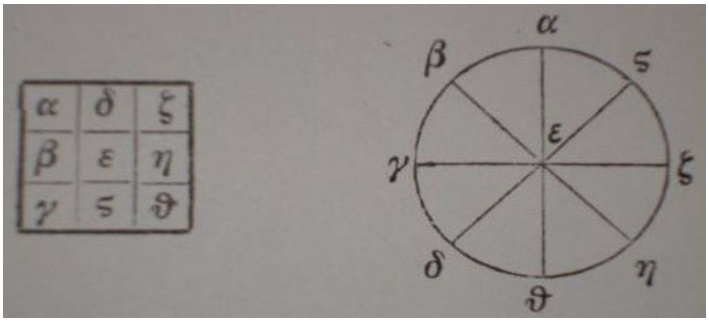
A corresponding transcription into Staff Notation of the above musical example – written in Byzantine Notation – could be formed as follows:



2. Ascending & Descending Full ‘Chromatic’ Scale (‘Octachord’ [Eighth]):



The present author would like to take this opportunity to finally mention that if one were to seek another approach, understanding, description and teaching of the Theory of the Greek Religious Music, beyond the notion of the circular shape, the answer is dictated, written, and structured once again by Theon of Smyrna, in his timeless work *Mathematics Useful for Understanding Plato.* It seems that no one has yet given any consideration or commented in particular the representation found in the “Properties of the Numbers Contained in the Decad (*Tetraktys*),” through two schemes (through two shapes: a circle and a square):



Diagrams of the Mathematical Wheel & Square of Theon of Smyrna.[25]

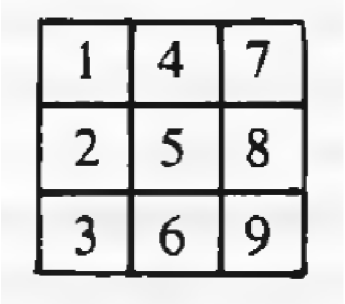
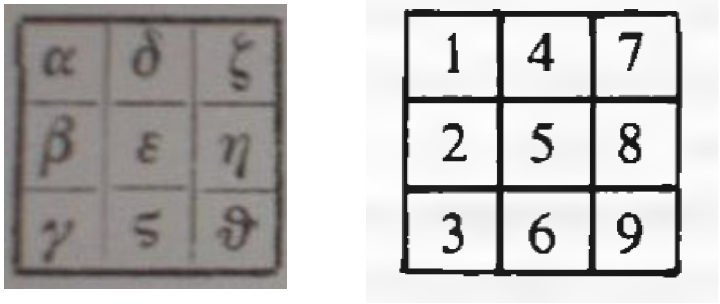


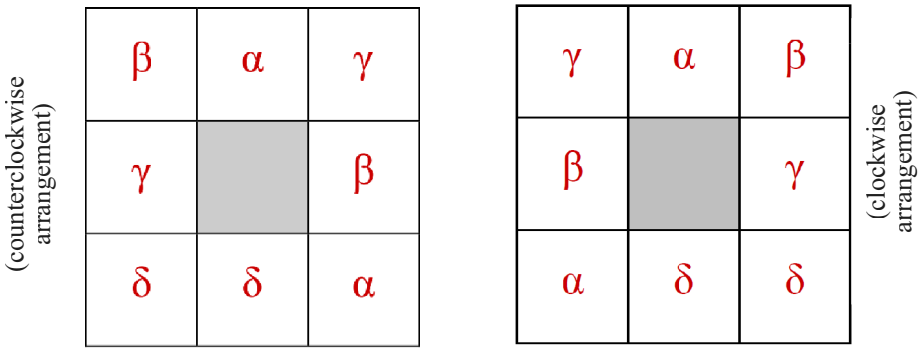
Diagram of the Mathematical Square of Theon of Smyrna.[26]

Everything regarding the circle has been commented above; concerning the second shape mentioned, namely, the square, it has to be noted that it is a square which is divided in nine equal smaller squares, a shape which can also be read counterclockwise in a serpentine manner. Any data that arise as an idea and philosophy from this shape could easily reflect, but in a square shaped representation, the basic data of the Theory of the Greek Religious Music usually provided cyclically. In other words, this notion could transform the known *Wheel of the Eight Modes of the Greek Religious Music* into a *Square of the Eight Modes of the Greek Religious Music*:

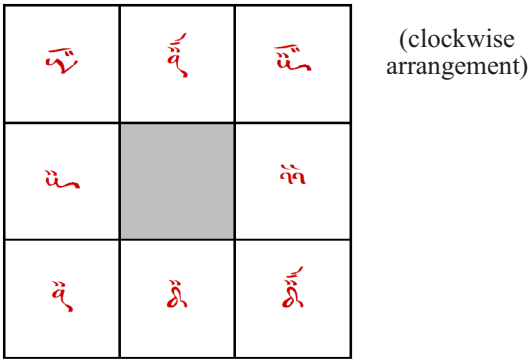
[25] Hiller 1878, 102.
[26] Lawlor/Toulis 1979, 67.



The Mathematical Square of Theon of Smyrna.

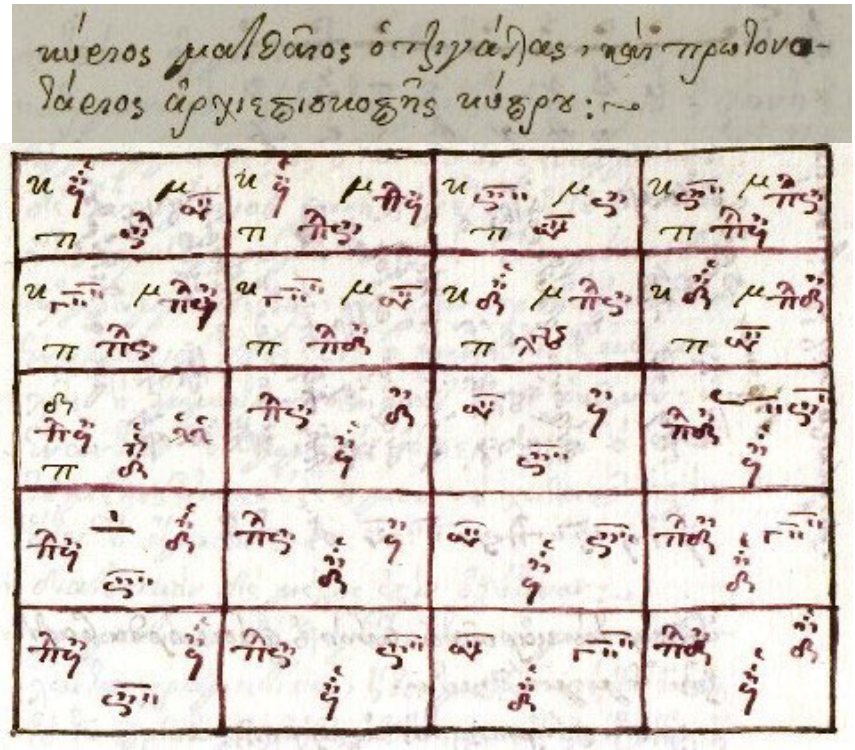


The Square of the Eight Modes of the Greek Religious Music
('arithmetically' sketched).



The Square of the Eight Modes of the Greek Religious Music
('musicologically' sketched).

A similar undertaking could significantly facilitate or even create new, unexpected and, until now, undiscovered perspectives of knowledge approach to such educational and tutorial schemes of teaching of the Theory of the Greek Religious Music, schemes such as, indicatively, the following one, that, in this case, is a *Parallelogram of the Eight Modes of the Greek Religious Music*, described in Greek terms as 'kanonion' (means theoretical norm):[27]



Codex No. 35/188[a] of K. Psachos's Collection (A.D. 1781), f. 114[r]
['Kanonion' made by the Protonotarios of Cyprus Archdiocese Matthew Tzigalas].

To sum up, it seems that it is about time to try to understand and grasp the deepest dimensions found within the shape of the *Wheel* (or even within the shapes of the *Square* or the *Parallelogram*); dimensions

[27] Cf. Chaldæakes 2020, 13–50.

and perspectives already given by Plato and commented by Theon of Smyrna; dimensions and perspectives reflected in the inner sense of the entire Greek Religious Music Theory. Goethe used to have a most appropriate quotation, which adequately fits for closing the present paper: "The more someone knows, the more one understands, the more one realizes that everything revolves in a circle."[28] *

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[28] Neville 2006, 265.

* Preliminary thoughts of those given in an expanded way in this paper have already been noticed by the present author in Chaldæakes 2014, 135–172.

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